Joint Spectrum Reservation and On-Demand Request for Mobile Virtual Network Operators

Yingxiao Zhang, Suzhi Bi, *Member, IEEE*, and Ying-Jun Angela Zhang, *Senior Member, IEEE*

***Abstract*—Wireless network virtualization enables mobile virtual network operators (MVNOs) to develop new services on a low-cost platform by leasing virtual resources from mobile network owners. In this paper, we investigate a two-stage spectrum leasing framework, where an MVNO acquires spectrum resources through both advance reservation and on-demand request. To maximize its surplus, the MVNO needs to jointly optimize the amount of spectrum resources to lease in the two stages by taking into account traffic intensity, random user locations, wireless channel statistics, quality-of-service requirements, and the price differences. Meanwhile, to maximize the utilization of the acquired resources, the MVNO dynamically allocates the spectrum resources to its mobile subscribers (users) according to fast wireless channel fading. We formulate the MVNO's surplus maximization problem as a tri-level nested optimization problem consisting of dynamic resource allocation (DRA), on-demand request, and advance reservation subproblems. To solve the problem efficiently, we first analyze the DRA problem, and then use the optimal solution to find the optimal leasing decisions in the two stages. In particular, we derive a closed-form expression of the optimal on-demand request, and develop a stochastic gradient descent algorithm to find the optimal advance reservation. For a special case when the proportional fairness utility function is adopted, we show that the optimal two-stage leasing scheme is related to the number of users and is irrelevant to user locations. Simulation results show that the two-stage spectrum leasing scheme can adapt to different levels of traffic and on-demand price variations, and achieve higher surplus than conventional one-stage leasing schemes.**

***Index Terms*—Network slicing, radio spectrum management, mobile virtual network operator, stochastic programming.**

I. Introduction

WIRELESS network virtualization (WNV) (or network slicing in 5G systems) is an emerging technology that provides unprecedented opportunities for mobile virtual network operators (MVNOs) to develop new services by leasing infrastructure and radio resources from mobile network owners (MNOs) [1]–[4]. In contrast to the network function virtualization in upper layers, the virtualization of radio spectrum in the physical layer is a unique problem in WNV, due to the broadcasting and stochastic nature of wireless channels [5]. Unlike other network resources, radio spectrum can be dynamically reused by different links based on the geographic separation between transmission nodes, the transmit powers, the interference cancellation capability, and the quality-of-service requirements. As a result, spectrum virtualization is much more complicated and deserves in-depth study.

Existing approaches for radio spectrum virtualization can be broadly classified into two categories: advance reservation and on-demand request. In advance reservation, each MVNO reserves an amount of spectrum resources from an MNO for a long period of time [6]–[9]. In particular, [6] considered four types of contract-based reservations and developed a hypervisor to allocate resources according to the predefined contracts. Reference [7] developed a network substrate to virtualize wireless resources, which enables both bandwidth-based and rate-based reservations. Reference [8] developed a partial resource reservation scheme, where each MVNO reserves a minimum amount of spectrum and the remaining part is shared among all MVNOs based on their real-time demands. Reference [9] proposed a dynamic resource allocation scheme, which keeps track of both the minimum reservations and the fairness requirement. In general, advance reservation is convenient for the MNO to pre-plan resource slicing and allocation. However, due to the uncertainty of traffic realizations and wireless channel fading, under-reservation (over-reservation) may occur when the reserved resource is less (more) than the real-time demands. In contrast, on-demand request preserves flexibility for MVNOs to order spectrum resources according to the observed traffic demands in real time [10]–[13]. For example, [10] considered virtualization in heterogeneous networks, where an MVNO leases resources from both macro and small cells for a particular realization of users. Reference [11] studied a cognitive MVNO, which not only leases spectrum resources from the MNO but also accesses the white space by spectrum sensing. Reference [12] developed a real-time trading mechanism under incomplete cost information from the MVNOs. Reference [13] derived a hierarchical combinatorial auction mechanism, where the leasing decisions vary as fast as the variations of instantaneous wireless channel fading. The flexibility of the on-demand

Manuscript received August 9, 2017; revised December 22, 2017 and February 12, 2018; accepted February 16, 2018. Date of publication February 27, 2018; date of current version July 13, 2018. This work was supported in part by General Research Funding from the Research Grants Council of Hong Kong under Project 14209414 and Project 14200315. The work of S. Bi was supported in part by the National Natural Science Foundation of China under Project 61501303, the Foundation of Shenzhen City under Project JCYJ20160307153818306, and the Science and Technology Innovation Commission of Shenzhen under Project 827/000212. This paper was presented in part at the 2016 IEEE ICCS, Shenzhen, China [16]. The associate editor coordinating the review of this paper and approving it for publication was S. Coleri Ergen. *(Corresponding author: Suzhi Bi.)*

Y. Zhang and Y.-J. A. Zhang are with the Department of Information Engineering, The Chinese University of Hong Kong, Hong Kong (e-mail: zy012@ie.cuhk.edu.hk; yjzhang@ie.cuhk.edu.hk).

S. Bi is with the College of Information Engineering, Shenzhen University, Shenzhen 518060, China (e-mail: bsz@szu.edu.cn).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TCOMM.2018.2809724

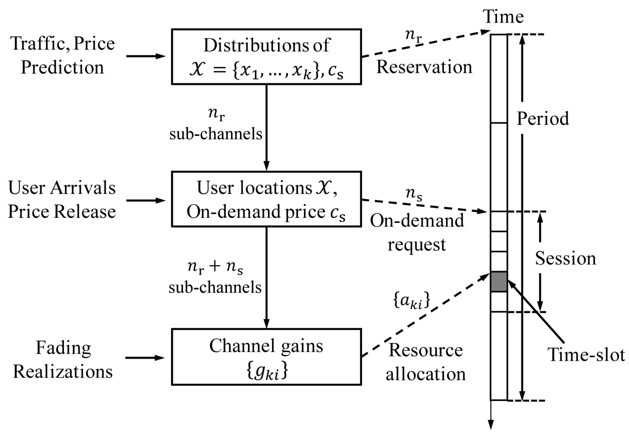


Fig. 1. Illustration of the proposed two-stage spectrum leasing framework.

request comes at a high operational cost of frequent calculation and realization of the leasing decisions. Moreover, there is no guarantee of sufficient spectrum supply for the MVNOs in real time.

In contrast to conventional one-stage leasing schemes, i.e., either advance reservation or on-demand request, this paper proposes a two-stage spectrum leasing scheme, where an MVNO acquires spectrum from both advance reservation and on-demand request. When carefully optimized, the two-stage leasing scheme enjoys the complementary strengths of the two stages. The first stage, advance reservation, reduces the risk and the operation complexity for both MVNOs and MNOs. On one hand, advance reservation ensures the MVNOs a baseline amount of spectrum resources at a relatively low cost. On the other hand, it allows the MNOs to pre-plan resource slicing and system operation in an early stage. Meanwhile, the second stage, on-demand request, preserves flexibility and competition. On one hand, the MVNOs can acquire additional spectrum resources according to the real-time traffic demands, and thus avoid being overly conservative in the first stage. On the other hand, the MNO can derive more profit by setting a higher on-demand price according to the real-time competition intensity among multiple MVNOs. The two-stage leasing framework has been previously considered in cloud networks to serve uncertain demands of computing and storage resources [14], [15]. However, in WNV, random wireless channel fading introduces a new dimension of uncertainty, rendering it difficult for an MVNO to anticipate its need of spectrum resources in advance.

In this paper, we aim to find the optimal two-stage spectrum leasing scheme for an MVNO. Specifically, the system operation involves three timescales, as depicted in Fig. 1. In the first stage, i.e., advance reservation, the MVNO reserves a number of sub-channels (SCs) for a long period of time, which usually covers hours or days. The decision is optimized according to the estimated statistics of the user traffic and the on-demand price over the period. In the second stage, i.e., on-demand request, the MVNO decides whether to request additional SCs after observing the actual locations of users it needs to serve and the current on-demand price. The on-demand request changes with user arrivals, departures, and movements, and usually varies in a timescale of seconds. In contrast to [10], [11], [13], the on-demand request here does not vary with fast channel fading, and thus involves much lower operational complexity. Then, in the timescale of milliseconds, the MVNO dynamically allocates the SCs acquired from both advance reservation and on-demand request to the users according to instantaneous wireless channel fading. To maximize the MVNO's profit, we need to jointly optimize the operations in all three timescales.

The two-stage spectrum leasing problem is naturally formulated as a tri-level nested optimization problem, which consists of dynamic resource allocation (DRA), on-demand request, and advance reservation subproblems. Solving the problem is challenging in two aspects. First, the nested structure makes the three subproblems closely intertwined. The decision made in a larger timescale affects the optimization in a smaller timescale. In turn, the optimal value of a smaller-timescale problem is embedded in the objective function of a larger-timescale problem. Hence, it is critical to analytically characterize the optimal values of the smaller-timescale problems, so that the larger-timescale problems are amenable to efficient solution algorithms. Secondly, the nested optimization problem is stochastic in the sense that the decision in a larger timescale must be optimized for random network realizations in a smaller timescale. As a first attempt, we have tried to solve the two-stage spectrum leasing problem in a precedent conference paper [16], which assumes α-fair utility functions and constant on-demand price. In this paper, we take into account the randomness of the on-demand price and extend the analysis to more general utility functions.

In this paper, we address the challenges as follows:

- We derive the optimal channel-aware DRA policy under a broad class of utility functions. Through rigorous analysis, we characterize the optimal utility as a function of the total number of SCs acquired from both advance reservation and on-demand request.
- Based on the result from DRA, we solve the on-demand request and advance reservation subproblems for general utility functions. In particular, we derive a closed-form expression of the optimal number of SCs to lease in the on-demand request stage. Moreover, we develop a stochastic gradient descent (SGD) algorithm to find the number of SCs to lease in the advance reservation stage.
- For the special case when proportional fairness (PF) utility function is adopted, we derive closed-form expressions of the optimal numbers of SCs to lease in the two stages. In other words, the MVNO can find the optimal operations in all three timescales by analytical calculations with negligible computational complexity.

Our numerical results show that the derived two-stage leasing scheme can exploit the on-demand price variations to reduce the MVNO's operating cost. Moreover, it can adapt to different levels of user traffic variations. The two-stage leasing scheme achieves more surplus than conventional one-stage spectrum leasing schemes.

The rest of paper is organized as follows. In Section II, we describe the network model and introduce the two-stage

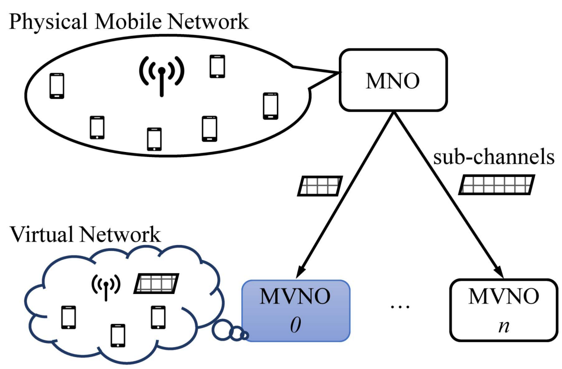


Fig. 2. A snapshot of wireless network virtualization. An MVNO leases sub-channels (SCs) from the MNO and programs on the acquired SCs to serve its users.

spectrum leasing framework. We solve the DRA problem for general utility functions in Section III. Then, we derive the optimal solution of the on-demand request and the advance reservation subproblems in Section IV. Further, in Section V, we analyze the special properties of the two-stage leasing scheme when PF utility is adopted. The numerical results and discussions are presented in Section VI. Finally, we conclude this work in Section VII.

II. System Model and Problem Formulation

Consider a single MVNO that acquires radio spectrum from an MNO and programs on the acquired spectrum to serve its users, as shown in Fig. 2. In particular, we focus on the downlink transmission in a single OFDMA cell, where the spectrum owned by the MNO is divided into a number of sub-channels (SCs). Without loss of generality, we assume that the cell is a circular area with radius D around a base station (BS) at the origin, denoted by $\mathcal{D}(o, D)$. A set of users requesting services from the MVNO at the same time is denoted by a random point process $\mathcal{X} = \{x_k, \forall k = 1, \dots, K\}$, where each $x_k \in \mathcal{D}$ is the location of the k-th user, and $K \in \mathbb{Z}_+$ is the total number of users. We assume that each x_k is uniformly distributed in the cell, and the distribution of K can be estimated accurately from historical data. The user set $\mathcal{X}$ changes with user arrivals, departures, or movements. In this way, the traffic statistics is characterized by the distribution of $\mathcal{X}$. We assume that both the BS and users are equipped with single antennas. Suppose that transmitted signals are affected by both large-scale path loss and fast Rayleigh fading. The instantaneous data rate for User k on SC i is given by

$$b_{ki} = B \log \left(1 + \frac{P\ell(\|x_k\|)g_{ki}}{\Gamma N_0 B}\right), \tag{1}$$

where B denotes the bandwidth of each SC, P denotes the fixed transmit power of the BS, $\ell(\cdot)$ denotes the path-loss function, $\|\cdot\|$ denotes the Euclidean norm, g_{ki} denotes the power gain of the i.i.d. Rayleigh fading, N_0 denotes the power spectral density of white noise at the receiver, and Γ denotes the capacity margin [17].

The two-stage spectrum leasing scheme involves operations in three timescales, as depicted in Fig. 1. For the sake of clarity, we define *a period, a session, and a time-slot* as three basic time units, during which traffic statistics (i.e., the distribution of $\mathcal{X}$), user locations (i.e., the realization of $\mathcal{X}$), and channel fading (i.e., $\{g_{ki}, \forall k, i\}$) remain unchanged, respectively. Typically, a period is measured in hours, a session in seconds, and a time-slot in milliseconds. At the beginning of a period, the MVNO reserves a number of SCs for the whole period according to the distribution of $\mathcal{X}$. Then, at the beginning of each session, the MVNO decides whether to request additional SCs for the session according to the observed realization of $\mathcal{X}$. The acquired SCs from both reservation and on-demand request are dynamically allocated to the users according to fast channel fading in each time-slot. The MVNO's surplus maximization problem can be formulated as a tri-level nested optimization problem, which consists the following three subproblems.

1) Advance Reservation: Let c_{r} be the reservation price of SCs, and n_{r} be the number of reserved SCs. The problem of finding the optimal advance reservation is formulated as

$$\underset{n_{\mathrm{r}} \in \mathbb{Z}_+}{\text{maximize}} \;\; -c_{\mathrm{r}} n_{\mathrm{r}} + \mathbf{E}_{\mathcal{X}, c_{\mathrm{s}}}[Q(\mathcal{X}, c_{\mathrm{s}}, n_{\mathrm{r}})], \tag{2}$$

where the first term in the objective function is the reservation cost, and the second term is the expected surplus from each session. The expectation is taken over all possible user realizations $\mathcal{X}$ and the on-demand price c_{s} in a session. Here, $Q(\mathcal{X}, c_{\mathrm{s}}, n_{\mathrm{r}})$ denotes the surplus obtained from a session with $\mathcal{X}$, c_{s} and n_{r}, and will be defined explicitly in the on-demand reservation subproblem. We assume that the reserved SCs cannot be released back to the MNO until the end of the period.

The MVNO's advance reservation should take into account the traffic uncertainty from the user side, as well as the price uncertainty from the MNO side. Specifically, the user set $\mathcal{X}$ varies across sessions due to random user arrivals, departures, and mobility. Moreover, the on-demand price c_{s} varies according to the competition intensity among MVNOs in each session. We assume that the MVNO can estimate the distribution of c_{s} during a period according to historical information. Let $F_{c_{\mathrm{s}}}(\cdot)$ and $f_{c_{\mathrm{s}}}(\cdot)$ denote the cumulative distribution function (CDF) and probability density function (PDF) of c_{s}, respectively. Note that constant c_{s} as has been studied in [16] is a special case where the variance of c_{s} is zero. Typically, the MNO sets c_{r} lower than c_{s} in order to encourage reservation in advance. We will discuss the impact of the price differences in details through rigorous analysis in the following sessions.

2) On-Demand Request: At the beginning of each session, the MVNO decides whether to request additional SCs according to the observed user set $\mathcal{X}$ and the on-demand price c_{s} announced by the MNO. Let n_{s} be the number of SCs to request in the session. The on-demand request problem is formulated as

$$Q(\mathcal{X}, c_{\mathrm{s}}, n_{\mathrm{r}}) = \underset{n_{\mathrm{s}} \in \mathbb{Z}_+}{\text{maximize}} \;\; -c_{\mathrm{s}} n_{\mathrm{s}} + u_{\mathrm{g}} G(\mathcal{X}, n_{\mathrm{r}} + n_{\mathrm{s}}), \tag{3}$$

where u_{g} is a fixed scaler that converts the utility into monetary unit, and $G(\mathcal{X}, n)$ denotes the maximum utility achieved by serving the user set $\mathcal{X}$ with n SCs. The first term in the objective function of (3) is the cost of the on-demand SCs, and the second term is the utility-based income of serving the users. The optimal value $Q(\mathcal{X}, c_s, n_r)$ is referred to as the surplus from the session, which becomes part of the MVNO's total profit. Note that the number of SCs from the on-demand request changes across sessions.

3) Dynamic Resource Allocation: The maximum system utility $G(\mathcal{X}, n)$ of a session is achieved by physical layer DRA. In particular, the MVNO dynamically allocates $n = n_r + n_s$ SCs to the K users in $\mathcal{X}$ according to the fast channel fading in each time-slot. Benefiting from the independent channel variations across users and SCs, DRA can substantially improve the system utility due to multiuser diversity. Let $U(\bar{r})$ denote the utility function of each user, where $\bar{r}$ is the average throughput over the session. We assume $U(\bar{r})$ as a continuously differentiable increasing concave function, which captures the satisfaction level of the user when received throughput $\bar{r}$ [18]. By deploying different utility functions, the MVNO can balance the trade-off between throughput maximization and service fairness among users.

Let $g_{ki}[t]$ denote the fast fading channel gain between the BS and User k on SC i at the time-slot t, and $b_{ki}[t]$ denote the corresponding achievable data rate calculated by (1). Let $a_{ki}[t] \in [0, 1]$ denote the fraction of airtime of SC i allocated to User k at the time-slot t, and $A = \{a_{ki}[t], \forall k, i, t\}$ denote all allocation decisions in the session. Then, the average throughput of User k (for $k = 1, \ldots, K$) is given by

$$\bar{r}_k = \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{n} a_{ki}[t] b_{ki}[t], \tag{4}$$

where T is the total number of time-slots in a session.

The problem of finding the optimal SC allocation policy for all time-slots can be formulated as

$$G(\mathcal{X}, n) = \underset{A}{\text{maximize}} \sum_{k=1}^{K} U(\bar{r}_k) \tag{5a}$$

$$\text{subject to} \sum_{k=1}^{K} a_{ki}[t] = 1, \quad \forall i,\ t \tag{5b}$$

$$a_{ki}[t] \geq 0, \quad \forall k,\ i,\ t. \tag{5c}$$

In particular, we have $G(\emptyset, n) = 0$, meaning that the utility is 0 if the user set is empty. Note that $G(\mathcal{X}, n)$ may be negative if $U(\bar{r})$ represents the penalties. For example, when $U(\bar{r}) = -1/\bar{r}$, the DRA problem in (5) tries to minimize the overall transmission delay.

From the description above, we can see that the three optimization problems are nested. The optimal values of the problems in smaller timescales $Q(\mathcal{X}, c_s, n_r)$ and $G(\mathcal{X}, n)$ are embedded in the objective functions in larger timescales (2) and (3), respectively. In turn, the decisions in larger timescales n_r and n_s are parameters of the problems in smaller timescales (3) and (5).

In what follows, we first study the DRA problem (5) in Section III. With the results from DRA, we then derive the optimal on-demand request and advance reservation for general utility functions in Section IV. Further, we analyze the special properties of the two-stage leasing scheme when PF utility function is adopted in Section V.

III. Dynamic Resource Allocation

We first study the DRA problem in (5), which tries to maximize the utilization of the acquired SCs from both advance reservation and on-demand request according to fast channel fading in each time-slot.

A. Optimal DRA Policy

Let $\Lambda = \{\lambda_i[t], \forall i, t\}$ and $\mathcal{V} = \{\nu_{ki}[t], \forall i, j, t\}$ denote the Lagrangian multipliers with respect to constraints (5b) and (5c), respectively. The Lagrange function is given by

$$L(A, \Lambda, \mathcal{V}) = \sum_{k=1}^{K} U(\bar{r}_k) + \sum_{i=1}^{n} \sum_{t=1}^{T} \lambda_i[t] \left(1 - \sum_{k=1}^{K} a_{ki}[t]\right) + \sum_{i=1}^{n} \sum_{k=1}^{K} \sum_{t=1}^{T} \nu_{ki}[t] a_{ki}[t]. \tag{6}$$

Let A^* denote the optimal solution and $\bar{\boldsymbol{r}}^* = [\bar{r}_1^*, \ldots, \bar{r}_K^*]$ denote the corresponding optimal average throughputs for all users. The following KKT conditions hold:

$$\left.\frac{\partial L}{\partial a_{ki}[t]}\right|_{A^*} = \nabla U(\bar{r}_k^*) \frac{b_{ki}[t]}{T} - \lambda_i[t] + \nu_{ki}[t] = 0, \quad \forall k,\ i,\ t \tag{7a}$$

$$\sum_{k=1}^{K} a_{ki}^*[t] = 1, \quad \forall i,\ t \tag{7b}$$

$$a_{ki}^*[t] \geq 0, \quad \nu_{ki}[t] \geq 0, \ \ \nu_{ki}[t] a_{ki}^*[t] = 0, \quad \forall k,\ i,\ t. \tag{7c}$$

Here, $\nabla U(r_0) = \frac{dU(\bar{r})}{d\bar{r}}\big|_{\bar{r}=r_0}$ is the first-order derivative of $U(\bar{r})$ evaluated at $\bar{r} = r_0$.

We can infer from (7a) and (7c) that if $a_{ki}^*[t] > 0$, then $\nu_{ki}[t] = 0$. Hence, we have

$$\nabla U(\bar{r}_k^*) b_{ki}[t] = T\lambda_i[t] \geq \nabla U(\bar{r}_{k'}^*) b_{k'i}[t], \quad \forall k' \neq k. \tag{8}$$

In other words, SC i is allocated to the user (users) that has (have) the largest value of $\nabla U(\bar{r}_k^*) b_{ki}[t]$. As $b_{ki}[t]$ is drawn from a continuous distribution, the probability that two or more users have the same value of $\nabla U(\bar{r}_k^*) b_{ki}[t]$ is zero. Therefore, each SC is exclusively allocated to a single user in each time-slot. The optimal SC allocation policy is described by:

$$a_{ki}^*[t] = \begin{cases} 1, & k = \arg\max_{k'} \nabla U(\bar{r}_{k'}^*) b_{k'i}[t] \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

The DRA policy in (9) states that an SC in each time-slot is allocated to the user which can obtain a relatively high bit rate with respect to a function of its average throughput. Note that the SC allocation policy in (9) requires the knowledge of the optimal average throughputs $\bar{\boldsymbol{r}}^*$, which will be derived in the next subsection.

B. Optimal Average Throughput

With the SC allocation policy in (9), the average throughput of each user is given by

$$\bar{r}_k^* = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{n} b_{ki}[t]\mathbf{1}_{\left\{\nabla U(\bar{r}_k^*)b_{ki}[t]>\nabla U_{k'}(\bar{r}_{k'}^*)b_{k'i}[t],\quad \forall k'\neq k\right\}}, \tag{10}$$

$\forall k = 1, \ldots, K$, where $\mathbf{1}_{\{\cdot\}}$ is the indicator function, which equals 1 if the argument is true and 0, otherwise. By the assumption that T is sufficiently large so that the channel fading process is ergodic, the time-averaged throughput in (10) can be transfered to the expectation with respect to the independent fast fading, which is given by

$$\begin{aligned}\bar{r}_k^* &= \sum_{i=1}^{n}\mathbf{E}_{\{b_{ki}\}}\left[b_{ki}\mathbf{1}_{\left\{\nabla U(\bar{r}_k^*)b_{ki}>\nabla U_{k'}(\bar{r}_{k'}^*)b_{k'i},\quad \forall k'\neq k\right\}}\right]\\ &= \sum_{i=1}^{n}\mathbf{E}_{b_{ki}}\left[b_{ki}\mathbf{Pr}\left(b_{k'i} < \frac{\nabla U(\bar{r}_k^*)}{\nabla U(\bar{r}_{k'}^*)}b_{ki},\quad \forall k'\neq k \middle| b_{ki}\right)\right]\\ &= \sum_{i=1}^{n}\mathbf{E}_{b_{ki}}\left[b_{ki}\prod_{k'\neq k}\mathbf{Pr}\left(b_{k'i} < \frac{\nabla U(\bar{r}_k^*)}{\nabla U(\bar{r}_{k'}^*)}b_{ki}\right)\right],\end{aligned} \tag{11}$$

where the last step follows from the independence of fast fading across users. Note that the division by $\nabla U(r)$ requires $\nabla U(r) > 0$, which is true for continuously increasing utility functions. As fast fadings are independently and identically distributed, b_{ki} for all i follow the same distribution, where the CDF $F_{b_k}(\cdot)$ and PDF $f_{b_k}(\cdot)$ can be calculated from the distribution of Rayleigh fading according to (1). Note that $F_{b_k}(\cdot)$ and $f_{b_k}(\cdot)$ only depend on the user's location x_k. Then, (11) can be written as

$$\begin{aligned}\bar{r}_k^* &= n\mathbf{E}_{b_k}\left[b_k\prod_{k'\neq k}F_{b_{k'}}\left(\frac{\nabla U(\bar{r}_k^*)}{\nabla U(\bar{r}_{k'}^*)}b_k\right)\right]\\ &= n\int_0^\infty \eta\prod_{k'\neq k}F_{b_{k'}}\left(\frac{\nabla U(\bar{r}_k^*)}{\nabla U(\bar{r}_{k'}^*)}\eta\right)f_{b_k}(\eta)\,d\eta.\end{aligned} \tag{12}$$

From (12), we can see that the optimal throughput $\bar{\boldsymbol{r}}^*$ is the root of the following non-linear equation system

$$r_k = n\Phi_k(\boldsymbol{r}),\quad \forall k = 1, \ldots, K, \tag{13}$$

where

$$\Phi_k(\boldsymbol{r}) = \int_0^\infty \eta\prod_{k'\neq k}F_{b_{k'}}\left(\frac{\nabla U(r_k)}{\nabla U(r_{k'})}\eta\right)f_{b_k}(\eta)\,d\eta. \tag{14}$$

By Brouwer's fixed-point theorem [19], the system (13) has at least one root. Moreover, from (14), we can see that $\Phi_k(\boldsymbol{r})$ is a decreasing function of r_k, since $\nabla U(r_k)$ is a decreasing function of r_k. This means that there is at most one r_k satisfying $r_k = n\Phi_k(\boldsymbol{r})$ given that all $\{\bar{r}_{k'}, \forall k' \neq k\}$ are fixed. Therefore, the equation system in (13) has a unique root.

The average throughputs of all users achieved by the optimal DRA policy can be calculated numerically by solving the non-linear system (13). Substituting the optimal average throughputs $\bar{\boldsymbol{r}}^*$ into (9), we can obtain the optimal DRA policy for each set of users that maximizes the system total utility with n SCs. Note that the DRA policy is computed at the beginning of each session according to the distribution of fast fading. Then, at the beginning of each time-slot, the SCs are allocated to serve users according to the estimated instantaneous channel fading.

C. Optimal System Utility

With the optimal DRA policy, we now derive the analytical expression of the optimal value $G(\mathcal{X}, n)$, which is required to solve the on-demand request problem in (3). With a bit abuse of notation, we use $\bar{\boldsymbol{r}}^*(n)$ to denote the optimal throughput vector achieved with n SCs. By solving the equation system (13), we can obtain $\bar{\boldsymbol{r}}^*(n)$ and the corresponding optimal utility $G(\mathcal{X}, n) = \sum_{k=1}^{K} U(\bar{r}_k^*(n))$. In general, there is no closed-form expression of $G(\mathcal{X}, n)$. To preserve analytical tractability, we focus on a broad class of utility functions, where the achieved average throughput increases linearly with the number of SCs.

Lemma 1: The average throughput of each user achieved by the optimal DRA policy in (9) increases linearly with the number of SCs, i.e.,

$$\bar{r}_k^*(n) = n\bar{r}_k^*(1),\ \text{for}\ k = 1, \ldots, K, \tag{15}$$

if the utility function satisfies the condition

$$\frac{\nabla U(r_1)}{\nabla U(r_2)} = \frac{\nabla U(r_1/n)}{\nabla U(r_2/n)}, \tag{16}$$

for any non-negative rates $r_1, r_2 > 0$ and positive integer $n = 1, 2, \ldots$.

Proof: With the property in (16), the average rate in (12) can be written as

$$\bar{r}_k^*(n)/n = \Phi_k(\bar{\boldsymbol{r}}^*(n)) = \Phi_k(\bar{\boldsymbol{r}}^*(n)/n),\quad \text{for}\ k = 1, \ldots, K. \tag{17}$$

This means that $\bar{\boldsymbol{r}}^*(n)/n$ for all n are roots of the equation system $r_k = \Phi_k(\boldsymbol{r})$ for $k = 1, \ldots, K$. As the system has a unique root $\bar{\boldsymbol{r}}^*(1)$, we have

$$\bar{\boldsymbol{r}}^*(n)/n = \bar{\boldsymbol{r}}^*(1),\quad \forall n = 1, \ldots, \tag{18}$$

which leads to the result in (15). ∎

The most well-known class of utility functions that satisfies condition (16) is the α-fair utility functions, which are defined by [20]

$$U(\bar{r}) = \begin{cases}\dfrac{\bar{r}^{(1-\alpha)}}{1-\alpha}, & \text{if}\ \alpha \geq 0,\ \alpha \neq 1,\\ \log(\bar{r}), & \text{if}\ \alpha = 1,\end{cases} \tag{19}$$

where α is the degree of fairness and $\bar{r}$ is the average throughput of a user. Specifically, the DRA problem based on α-fair utility turns out to be a throughput maximization problem when $\alpha = 0$, and becomes a delay minimization problem when $\alpha = 2$. Moreover, when $\alpha = 1$, proportional fairness is achieved among the users with the logarithm utility function. Note that not all increasing concave utility functions satisfy condition (16). For example, the exponential utility function with $U(\bar{r}) = 1 - e^{-\bar{r}}$ and the positive diminishing return with $U(\bar{r}) = \ln(1+\bar{r})$ [21].

When (16) holds, the average user throughput achieved by the optimal DRA policy is a linear function of the number of SCs. As a result, we only need to solve the non-linear system (13) once for $n = 1$, and then the average throughputs for a general n are obtained accordingly. The optimal utility is given by

$$G(\mathcal{X}, n) = \sum_{k=1}^{K} U(\bar{r}_k^*(n)) = \sum_{k=1}^{K} U(n\bar{r}_k^*(1)), \tag{20}$$

which will be used to derive the optimal on-demand request and advance reservation in the next section. Note that $\bar{\boldsymbol{r}}^*(1)$ is uniquely determined by the user locations $\mathcal{X}$ and the distribution of fast channel fading.

IV. Optimal Two-Stage Leasing Scheme

In this section, we investigate the optimal on-demand request and advance reservation for a broad class of utility functions that satisfy the condition in Lemma 1.

A. Optimal On-Demand Request

With the optimal utility obtained from the DRA problem, the on-demand request problem in (3) becomes

$$Q(\mathcal{X}, c_s, n_r) = \max_{n_s \in \mathbb{Z}_+} -c_s n_s + u_g \sum_{k=1}^{K} U\left((n_r + n_s)\bar{r}_k^*(1)\right). \tag{21}$$

By the property (16), we can compute the first-order derivative of the objective function with respect to n_s as

$$\begin{aligned}
&-c_s + u_g \sum_{k=1}^{K} \bar{r}_k^*(1) \nabla U\left((n_r + n_s)\bar{r}_k^*(1)\right) \\
&\quad = -c_s + u_g \nabla U(n_r + n_s) \sum_{k=1}^{K} \bar{r}_k^*(1) \frac{\nabla U\left((n_r + n_s)\bar{r}_k^*(1)\right)}{\nabla U(n_r + n_s)} \\
&\quad = -c_s + u_g \nabla U(n_r + n_s) \sum_{k=1}^{K} \bar{r}_k^*(1) \frac{\nabla U\left(\bar{r}_k^*(1)\right)}{\nabla U(1)} \\
&\quad = -c_s + u_g \nabla U(n_r + n_s)\Theta(\mathcal{X}),
\end{aligned} \tag{22}$$

where

$$\Theta(\mathcal{X}) = \frac{1}{\nabla U(1)} \sum_{k=1}^{K} \bar{r}_k^*(1) \nabla U\left(\bar{r}_k^*(1)\right). \tag{23}$$

Note that $\Theta(\mathcal{X})$ is uniquely determined by the user locations $\mathcal{X}$, and the value can be calculated numerically by solving the non-linear system (13) for $n = 1$. Since the objective in (21) is a concave function of n_s, setting the first-order derivative (22) to zero yields the optimal solution

$$n_s^* = \max\left(\nabla U^{-1}\left(\frac{c_s}{u_g \Theta(\mathcal{X})}\right) - n_r, 0\right), \tag{24}$$

where $\nabla U^{-1}(\cdot)$ is the inverse function of $\nabla U(\cdot)$. The integer solution can be obtained by rounding n_s^*. From (24), we can see that the MVNO uses the reserved n_r SCs to serve a baseline amount of traffic, and requests additional SCs when $c_s < u_g \Theta(\mathcal{X}) \nabla U(n_r)$. Due to the concavity of $U(\cdot)$, $\nabla U(\cdot)$ is a decreasing function, and hence n_s^* decreases with a higher c_s or a smaller $\Theta(\mathcal{X})$. This means that the MVNO requests less SCs when observing a higher on-demand price or a user realization with a smaller utility metric $\Theta(\mathcal{X})$.

A special case is the linear utility function, where $U(\bar{r}) = \bar{r}$. In this case, n_s^* is either zero or infinity, depending on $\Theta(\mathcal{X}) = \sum_{k=1}^{K} \bar{r}_k^*(1)$. This is because that both the leasing cost and the users' utility increase linearly with n_s, and the slope depends on $\Theta(\mathcal{X})$. In practice, the number of SCs for sale is usually limited. Hence, the optimal strategy for the MVNO is to lease as much SCs as possible if $c_s < u_g \Theta(\mathcal{X}) \nabla U(n_r)$, or no on-demand request, otherwise.

With the optimal on-demand request (24), we can compute the corresponding surplus in the session as

$$Q(\mathcal{X}, c_s, n_r) = \begin{cases} u_g \sum_{k=1}^{K} U(n_r \bar{r}_k^*(1)), & \text{if } c_s > u_g \Theta(\mathcal{X}) \nabla U(n_r), \\ \tilde{Q}^*(\mathcal{X}, c_s, n_r), & \text{otherwise}, \end{cases} \tag{25}$$

where

$$\begin{aligned}
\tilde{Q}^*(\mathcal{X}, c_s, n_r) &= c_s n_r - c_s \nabla U^{-1}\left(\frac{c_s}{u_g \Theta(\mathcal{X})}\right) \\
&\quad + u_g \sum_{k=1}^{K} U\left(\nabla U^{-1}\left(\frac{c_s}{u_g \Theta(\mathcal{X})}\right) \bar{r}_k^*(1)\right).
\end{aligned} \tag{26}$$

Specifically, we have $Q(\emptyset, c_s, n_r) = 0$, which corresponds to zero payoff in an idle session. Here, we use the real-value solution n_s^* as an approximation to the integer solution in calculating $Q(\mathcal{X}, c_s, n_r)$. The approximation error in calculating the optimal advance reservation in the next subsection is negligible, since the approximation error can be averaged out in $\mathbf{E}[Q(\mathcal{X}, c_s, n_r)]$.

B. Optimal Advance Reservation

With the expression of $Q(\mathcal{X}, c_s, n_r)$ in (25), we now solve the advance reservation problem in (2). Let $J(n_r)$ and $\nabla J(n_r)$ denote the objective function of (2) and its first-order derivative, respectively. With (16), we can compute $\nabla J(n_r)$ as

$$\nabla J(n_r) = -c_r + \mathbf{E}_{\mathcal{X}, c_s}\left[\min\left(c_s, u_g \Theta(\mathcal{X}) \nabla U(n_r)\right)\right]. \tag{27}$$

Proposition 1: When $\mathbf{E}[c_s] \leq c_r$, the MVNO makes no advance reservation, i.e., $n_r^* = 0$. In other words, all SCs are acquired from on-demand request in each session.

Proof: From (27), we have

$$\begin{aligned}
\nabla J(n_r) &= -c_r + \mathbf{E}_{\mathcal{X}, c_s}\left[\min\left(c_s, u_g \Theta(\mathcal{X}) \nabla U(n_r)\right)\right] \\
&\leq -c_r + \mathbf{E}[c_s],
\end{aligned} \tag{28}$$

due to the fact that $c_s \geq \min\left(c_s, u_g \Theta(\mathcal{X}) \nabla U(n_r)\right)$. When $\mathbf{E}[c_s] \leq c_r$, $\nabla J(n_r) \leq 0$, meaning that $J(n_r)$ is a non-increasing function of n_r. Therefore, the optimal non-negative solution is $n_r^* = 0$, which completes the proof. ■

Proposition 1 shows that a price discount is essential to motivate an MVNO to place a reservation in advance.

From the MNO's perspective, advance reservation has advantages of risk-free income and simple operation. Hence, the MNO usually sets a discount to encourage MVNOs to reserve resources for a long period of time.

For the case $\mathbf{E}[c_{\rm s}] > c_{\rm r}$, as $J(n_{\rm r})$ is a concave function of $n_{\rm r}$, the optimal real-value solution $n_{\rm r}^*$ satisfies the first-order condition

$$0 = -c_{\rm r} + \mathbf{E}_{\mathcal{X},c_{\rm s}}\left[\min\left(c_{\rm s}, u_{\rm g}\Theta(\mathcal{X})\nabla U(n_{\rm r}^*)\right)\right]. \quad (29)$$

From (29), we can see that $n_{\rm r}^*$ increases for a lower $c_{\rm r}$ or higher $c_{\rm s}$. This matches the intuition that the MVNO reserves more SCs in advance for a discounted reservation price such that it can avoid expensive on-demand request.

Due to the lack of closed-form expression of $\Theta(\mathcal{X})$, neither $J(n_{\rm r})$ nor $\nabla J(n_{\rm r})$ can be computed analytically even with the distribution functions of $c_{\rm s}$ and $\mathcal{X}$. Therefore, there is no analytical expression for the optimal reservation $n_{\rm r}^*$. As $J(n_r)$ is concave, we develop a stochastic gradient descent (SGD) algorithm, as presented in Algorithm 1, to find the optimal number of SCs to lease in the advance reservation stage. The key idea is to approximate the real gradient by that of a sampled user set $\mathcal{X}$ in each iteration, which is given by

$$\begin{aligned}\nabla_{\mathcal{X}} J(n_{\rm r}) &= -c_{\rm r} + \mathbf{E}_{c_{\rm s}}\left[\min\left(c_{\rm s}, u_{\rm g}\Theta(\mathcal{X})\nabla U(n_{\rm r})\right)\right] \\ &= -c_{\rm r} + \int_0^{u_{\rm g}\Theta(\mathcal{X})\nabla U(n_{\rm r})} \left(1 - F_{c_{\rm s}}(\eta)\right) d\eta, \quad (30)\end{aligned}$$

where the last step follows from that

$$\begin{aligned}\mathbf{E}_{c_{\rm s}}\left[\min(c_{\rm s}, c)\right] &= \int_0^c \eta dF_{c_{\rm s}}(\eta) + c\left(1 - F_{c_{\rm s}}(c)\right) \\ &= cF_{c_{\rm s}}(c) - \int_0^c F_{c_{\rm s}}(\eta)d\eta + c\left(1 - F_{c_{\rm s}}(c)\right) \\ &= \int_0^c \left(1 - F_{c_{\rm s}}(\eta)\right) d\eta. \quad (32)\end{aligned}$$

With the CDF of $c_{\rm s}$, the approximate gradient can be computed efficiently by (30). By [22, Th. 3.4], the SGD algorithm returns an ϵ-approximate solution after $L = O(1/\epsilon^2)$ iterations, when the step size is set to $\eta[l] = 1/\sqrt{l}$ for the l-th iteration step. The integer solution can be obtained by rounding the optimal real-value solution.

Algorithm 1 SGD to Find the Optimal Advance Reservation

Input: initial value $n_{\rm r}[0] \in \mathbb{Z}_+$, step size $\{\eta[l], 1 \le l \le L\}$

1: **for** $l = 1$ **to** L **do**
2: Sample a set of users, which have locations $\mathcal{X} = \{x_1, \dots, x_K\}$
3: Calculate the average throughputs $\bar{\boldsymbol{r}}^*(1)$ by solving (13) for $n = 1$, and the corresponding $\Theta(\mathcal{X})$ by (23)
4: Calculate the gradient $\nabla_{\mathcal{X}} J(n_{\rm r}[l-1])$ by (30)
5: Update and project :

$$\begin{aligned} n_{\rm r}[l] &= n_{\rm r}[l-1] + \eta[l]\nabla_{\mathcal{X}} J(n_{\rm r}[l-1]) \\ n_{\rm r}[l] &= \max(n_{\rm r}[l], 0) \end{aligned} \quad (31)$$

6: **end for**

Output: $n_{\rm r}^* = \frac{1}{L}\sum_{l=1}^L n_{\rm r}[l]$

V. Special Case: Proportional Fairness Utility

In this section, we apply the analytical results in the previous section to PF utility, and show the unique properties of the two-stage leasing scheme that are not reflected in that for general utility functions. Formally, the PF utility function is defined by $U(\bar{r}) = \log(\bar{r})$, and the first-order derivative is given by $\nabla U(\bar{r}) = 1/\bar{r}$, which satisfies the condition in Lemma 1 [23].

A. Optimal On-Demand Request

Substituting $\nabla U(\bar{r}) = 1/\bar{r}$ into (23), we get $\Theta(\mathcal{X}) = K$. Then, the optimal on-demand request in (24) can be simplified as

$$n_{\rm s}^* = \max\left(\frac{u_{\rm g}}{c_{\rm s}}K - n_{\rm r}, 0\right). \quad (33)$$

In contract to the general case (24), the optimal on-demand request for PF in (33) is only related to the number of users in the session and is irrelevant to the user locations. This is because that in the sum utility $G(\mathcal{X}, n) = K\log(n) + \sum_{k=1}^K \log\left(\bar{r}_k^*(1)\right)$, the information about user locations $\mathcal{X}$ only appears in the second additional term that is irrelevant to the leasing decision n. The physical meaning of (33) can be interpreted as follows. The MVNO needs no additional SCs if there are only a small number of active users or the on-demand price is very high, i.e., $u_{\rm g}K \le n_{\rm r}c_{\rm s}$. Otherwise, the MVNO requests additional SCs, whose number increases linearly with the number of users. Moreover, we can see that the MVNO's on-demand request is proportional to the inverse of $c_{\rm s}$, which is the same as the widely used demand curve in telecommunication systems [21].

B. Optimal Advance Reservation

Substituting $\nabla U(\bar{r}) = 1/\bar{r}$ and $\Theta(\mathcal{X}) = K$ into (29), we have

$$0 = -c_{\rm r} + \mathbf{E}_{K,c_{\rm s}}\left[\min\left(c_{\rm s}, u_{\rm g}K/n_{\rm r}^*\right)\right] \quad (34)$$

As shown in Proposition 1, $n_{\rm r}^* > 0$ for $\mathbf{E}[c_{\rm s}] > c_{\rm r}$. In this case, (34) can be rewritten as

$$\begin{aligned} c_{\rm r}n_{\rm r}^* &= \mathbf{E}_{K,c_{\rm s}}\left[\min\left(c_{\rm s}n_{\rm r}^*, u_{\rm g}K\right)\right] \\ &= \int \mathbf{Pr}\left(\min\left(c_{\rm s}n_{\rm r}^*, u_{\rm g}K\right) \ge \eta\right) d\eta \\ &= \int \left(1 - F_{c_{\rm s}}(\eta/n_{\rm r}^*)\right)\left(1 - F_K(\eta/u_{\rm g})\right) d\eta, \quad (35)\end{aligned}$$

where the first step follows that $\mathbf{E}[Y] = \int \mathbf{Pr}(Y \ge \eta)d\eta$ for a non-negative random variable Y, and $F_K(\cdot)$ is the CDF of K. With the CDFs of $c_{\rm s}$ and K, we can obtain $n_{\rm r}^*$ by solving the equation in (35). Note that $n_{\rm r}^*$ in the case of PF only depends on the distribution of K instead of the distribution of user locations $\mathcal{X}$ as in the case of general utility functions. In other words, the MVNO can easily compute the optimal number of SCs to lease in the advance reservation stage without applying Algorithm 1.

Proposition 2: In the case of PF utility function, the average cost of the optimal two-stage leasing scheme increases linearly with the average number of users in each session. That is

$$c_{\rm r}n_{\rm r}^* + \mathbf{E}[c_{\rm s}n_{\rm s}^*] = u_{\rm g}\mathbf{E}[K]. \quad (36)$$

Proof: From (34), we can compute the cost on the reserved SCs by

$$\begin{aligned} c_{\mathrm{r}} n_{\mathrm{r}}^* &= \mathbf{E}_{K,c_{\mathrm{s}}}\left[\min\left(c_{\mathrm{s}}, u_{\mathrm{g}} K/n_{\mathrm{r}}^*\right)\right] \cdot n_{\mathrm{r}}^* \\ &= \mathbf{E}_{K,c_{\mathrm{s}}}\left[\min(c_{\mathrm{s}} n_{\mathrm{r}}^*, u_{\mathrm{g}} K)\right]. \end{aligned} \tag{37}$$

Further, from (33), we can calculate the average cost on the on-demand request by

$$\begin{aligned} \mathbf{E}[c_{\mathrm{s}} n_{\mathrm{s}}^*] &= \mathbf{E}_{K,c_{\mathrm{s}}}\left[c_{\mathrm{s}} \cdot \max\left(\frac{u_{\mathrm{g}}}{c_{\mathrm{s}}} K - n_{\mathrm{r}}^*, 0\right)\right] \\ &= \mathbf{E}_{K,c_{\mathrm{s}}}\left[\max\left(u_{\mathrm{g}} K, c_{\mathrm{s}} n_{\mathrm{r}}^*\right) - c_{\mathrm{s}} n_{\mathrm{r}}^*\right]. \end{aligned} \tag{38}$$

Adding (37) to (38), we can obtain the total average cost as

$$\begin{aligned} & c_{\mathrm{r}} n_{\mathrm{r}}^* + \mathbf{E}[c_{\mathrm{s}} n_{\mathrm{s}}^*] \\ &= \mathbf{E}_{K,c_{\mathrm{s}}}\left[\min(c_{\mathrm{s}} n_{\mathrm{r}}^*, u_{\mathrm{g}} K) + \max(c_{\mathrm{s}} n_{\mathrm{r}}^*, u_{\mathrm{g}} K) - c_{\mathrm{s}} n_{\mathrm{r}}^*\right] \\ &= \mathbf{E}_{K,c_{\mathrm{s}}}\left[c_{\mathrm{s}} n_{\mathrm{r}}^* + u_{\mathrm{g}} K - c_{\mathrm{s}} n_{\mathrm{r}}^*\right] \\ &= u_{\mathrm{g}} \mathbf{E}[K], \end{aligned} \tag{39}$$

which leads to the proof. ■

Proposition V-B shows that more investment is needed for a busy period with a large number of users. However, the allocation of the investment into advance reservation and on-demand request depends on the variations of K and c_{s}, which will be discussed with numerical results in the next section.

VI. Numerical Results and Discussion

In this section, we evaluate numerically the performance of the derived two-stage leasing scheme. The users of a tagged MVNO are randomly dropped inside a circular cell with radius $D = 1$ km. The number of users in each session K is a random integer uniformly distributed in the interval $[0, 16]$. The wireless channel is Rayleigh faded. A standard path-loss model with path-loss exponent 3.67 is adopted [17]. The average received SNR at the cell edge is -6 dB. Suppose that the reservation price c_{r} is normalized to 1 and $u_{\mathrm{g}} = 5$. The on-demand price is uniformly distributed in $[0.8, 1.8]$, unless otherwise specified. We consider general α-fair utility functions with $\alpha = 1$ and $\alpha = 0.8$ as two examples. Notice that $\alpha = 1$ corresponds to PF and the results are given in Section V. Extensive simulations show that the use of other distributions or utility functions does not change the conclusions in this paper, and thus are omitted for brevity. We run system-level simulations using MATLAB software.

Fig. 3 shows the convergence process of Algorithm 1. The solution $n_{\mathrm{r}}[l]$ is plotted as a function of the iteration step l. The main complexity of Algorithm 1 is to sample enough realizations of user set, which can be obtained from historical records. Since the sampling is performed offline, the complexity of Algorithm 1 is not an issue for practical implementations. With the optimal advance reservation, the MVNO can calculate the optimal on-demand request in each session by the closed-form expression in (24). The overhead to acquire user locations and the on-demand price at the beginning of each session is very low compared with the duration of the session. The leased SCs from both advance reservation and on-demand request are allocated to the users following the optimal DRA policy in (9), where channel estimation has been standardized in physical layer protocols [17]. With our analytical results, the MVNO can implement two-stage leases and DRA with low operational complexity.

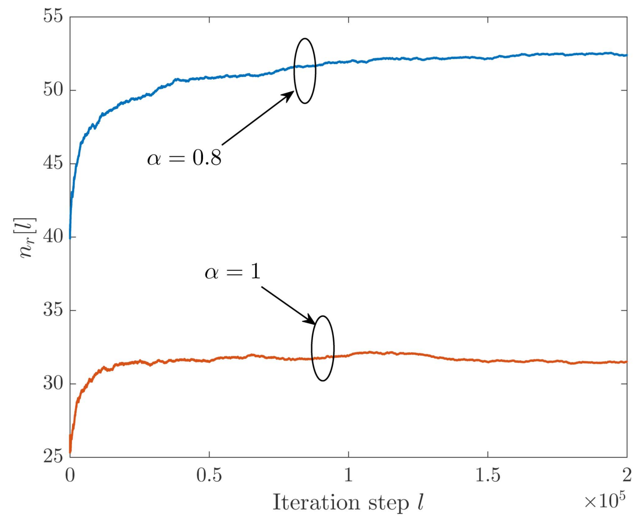


Fig. 3. Convergence process of the SGD algorithm to find the optimal reservation n_{r}^*.

As we have shown in Section IV, the optimal leasing decisions in the two stages depend on the variations of the on-demand price c_{s} and the user set $\mathcal{X}$. In this section, we will investigate the impact of the variations of c_{s} and $\mathcal{X}$ through numerical simulations. In particular, the variation level of a random variable z is measure by the coefficient of variation, which is defined by

$$\xi_z = \frac{\sigma_z}{\mu_z}, \tag{40}$$

where σ_z and μ_z are the standard deviation and the mean of the random variable z, respectively. With ξ_z, we can compare the variation levels of z under different scales.

A. Impact of On-Demand Price Variation

In Fig. 4, we fix the mean of the on-demand price $\mu_{c_{\mathrm{s}}}$, and vary the coefficient of variation $\xi_{c_{\mathrm{s}}}$ to investigate the impact of the on-demand price variation on the optimal two-stage leasing scheme. For uniform distribution, the support of c_{s} is given by $\left[(1-\sqrt{3}\xi_{c_{\mathrm{s}}})\mu_{c_{\mathrm{s}}}, (1+\sqrt{3}\xi_{c_{\mathrm{s}}})\mu_{c_{\mathrm{s}}}\right]$. In particular, $\xi_{c_{\mathrm{s}}} = 0$ corresponds to the case of constant c_{s} over the entire period. We focus on the case $\mu_{c_{\mathrm{s}}} > c_{\mathrm{r}}$ to avoid the trivial solution $n_{\mathrm{r}}^* = 0$ as shown in Proposition 1. Specifically, $\mu_{c_{\mathrm{s}}}$ is set to be 1.2 and 1.3 in the figures.

In Fig. 4a and 4b, we plot the optimal advance reservation n_{r}^* and the expectation of the optimal on-demand request $\mathbf{E}[n_{\mathrm{s}}^*]$ as functions of $\xi_{c_{\mathrm{s}}}$, respectively. We can see that as $\xi_{c_{\mathrm{s}}}$ becomes larger, n_{r}^* decreases and $\mathbf{E}[n_{\mathrm{s}}^*]$ increases, meaning that the MVNO places less reservation in advance and makes more on-demand request when the on-demand price has a larger variation. Moreover, this trend holds for utility functions with different fairness factors α and for different $\mu_{c_{\mathrm{s}}}$. For a larger $\mu_{c_{\mathrm{s}}}$, more SCs are reserved in advance and less SCs

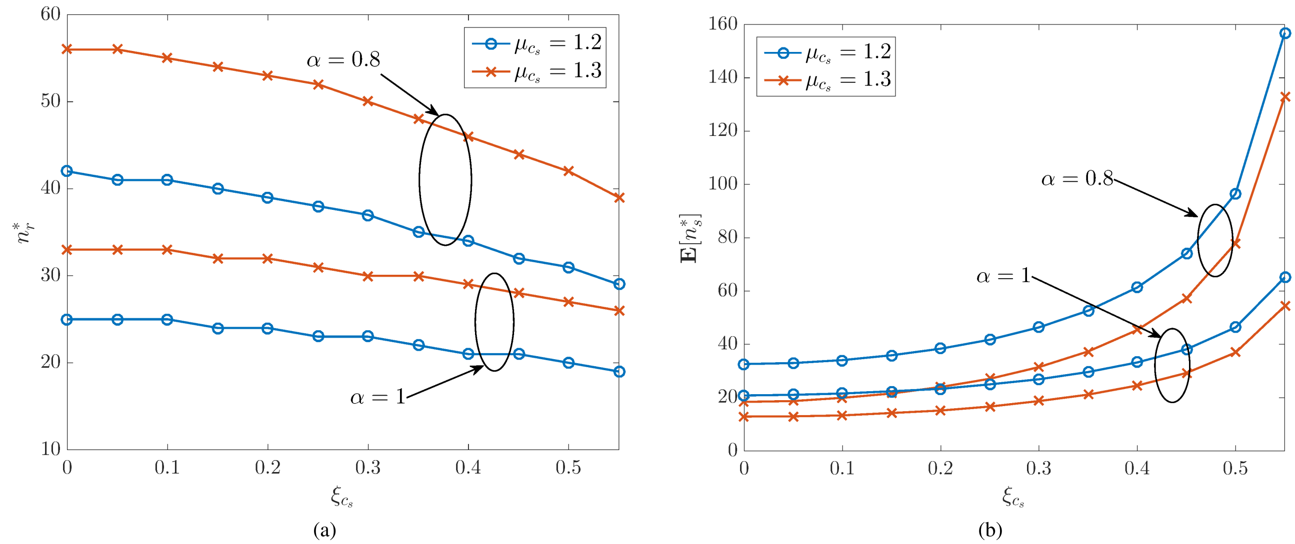


Fig. 4. Optimal two-stage leasing decisions versus on-demand price variation coefficient $\xi_{c_{\mathrm{s}}}$. (a) Optimal advance reservation n_{r}^*. (b) Expectation of the optimal on-demand request $\mathbf{E}\left[n_{\mathrm{s}}^*\right]$.

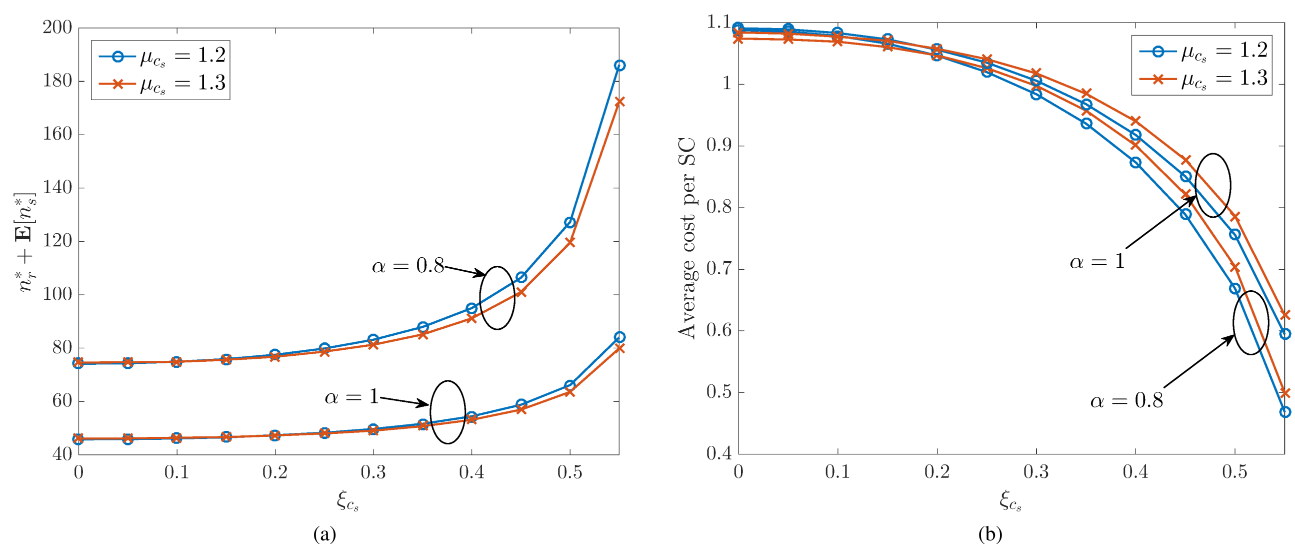


Fig. 5. Average amount of total leased SCs and the cost versus on-demand price variation coefficient $\xi_{c_{\mathrm{s}}}$. (a) Expected number of total leased SCs per session $n_{\mathrm{r}}^* + \mathbf{E}[n_{\mathrm{s}}^*]$. (b) Average cost per SC $\left(c_{\mathrm{r}} n_{\mathrm{r}}^* + \mathbf{E}[c_{\mathrm{s}} n_{\mathrm{s}}^*]\right) / \left(n_{\mathrm{r}}^* + \mathbf{E}[n_{\mathrm{s}}^*]\right)$.

are leased on demand due to the higher cost of on-demand request.

We further show the average total number of SCs used per session and the average leasing cost per SC in Fig. 5a and 5b, respectively. From Fig. 5a, we can see that more SCs are leased in total as $\xi_{c_{\mathrm{s}}}$ becomes larger. This is because that the lower percentile of c_{s} becomes lower when the variation is large. Sometimes, the realizations of c_{s} may be even lower than c_{r}, which provides the MVNO an opportunity to lease on-demand SCs at a low cost. Indeed, as shown in Fig. 5b, the average leasing cost per SCs becomes lower when $\xi_{c_{\mathrm{s}}}$ increases. This implies that the derived two-stage leasing scheme can exploit the short-term low on-demand price to reduce the MVNO's cost. As a result, more SCs are ordered in total for a larger $\xi_{c_{\mathrm{s}}}$. In addition, we can see that the total number of leased SCs increases and the average cost decreases for a lower $\mu_{c_{\mathrm{s}}}$. This matches the intuition that the MVNO consumes more SCs for a lower on-demand price.

B. Impact of Traffic Variation

One advantage of the two-stage leasing scheme is that the MVNO has the flexibility of adapting its leasing request according to the real-time traffic realizations during the on-demand request stage. In Fig. 6, we fix the mean of traffic intensity μ_K and vary the coefficient of variation ξ_K to investigate the impact of traffic variation on the optimal leasing decisions. From Fig. 6a and 6b, we can see that when ξ_K increases, the optimal reservation n_{r}^* decreases, and the expectation of the optimal on-demand request $\mathbf{E}[n_{\mathrm{s}}^*]$ increases. This is intuitive in the sense that the MVNO tends to rely more on the on-demand request while less on advance

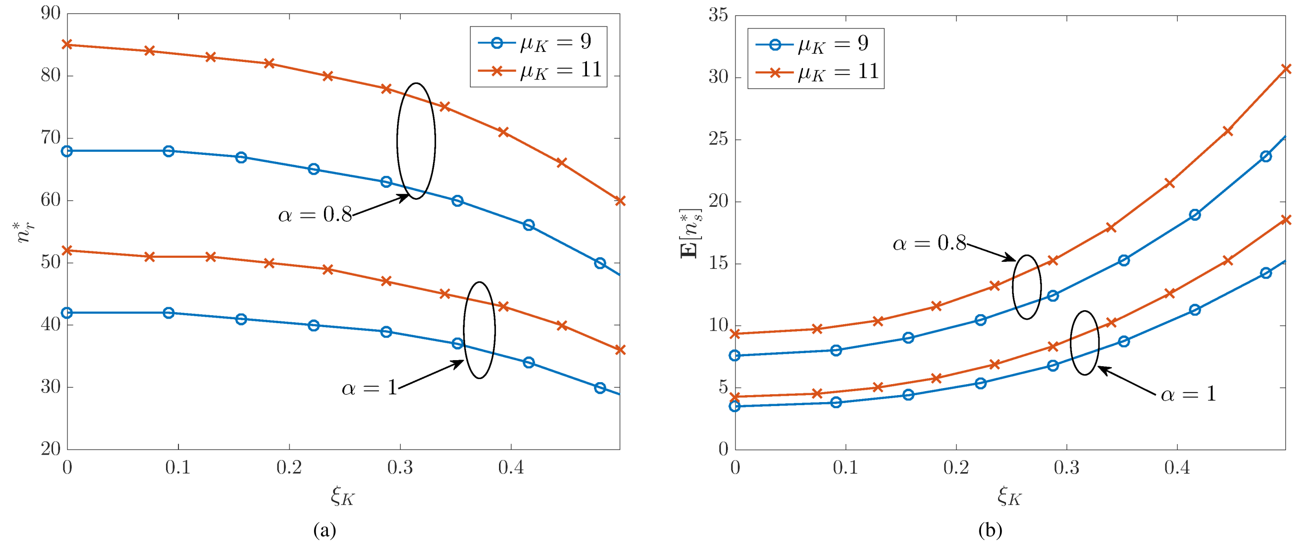


Fig. 6. Optimal two-stage leasing decisions versus traffic variation coefficient ξ_K. (a) Optimal advance reservation n_{r}^*. (b) Expectation of the optimal on-demand request $\mathbf{E}\left[n_{\mathrm{s}}^*\right]$.

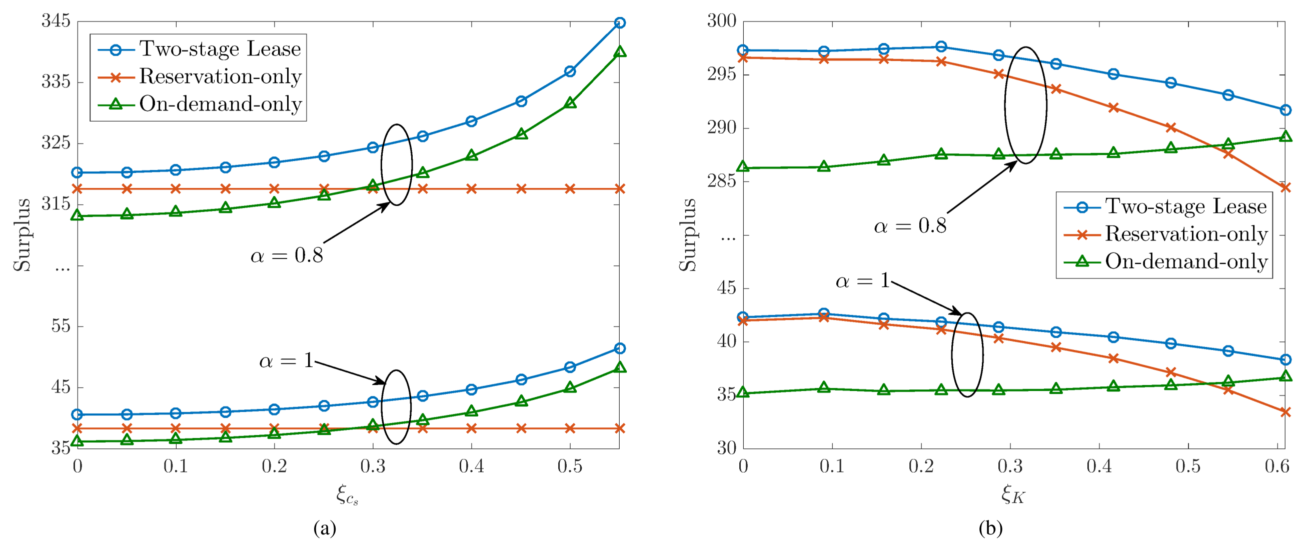


Fig. 7. Comparison of the two-stage leasing scheme with one-stage leasing schemes. (a) Surplus versus on-demand price variation coefficient $\xi_{c_{\mathrm{s}}}$. (b) Surplus versus traffic variation coefficient ξ_K.

reservation in order to handle more fluctuated traffic demands. Moreover, we can see that this observation applies to different traffic intensities μ_K and different fairness factors α. Further, we observe that more SCs are purchased in both advance reservation and on-demand request for a larger μ_K, which matches our intuition that the MVNO needs more SCs to serve more intense traffic.

C. Comparison With One-Stage Leasing Schemes

For comparison, we consider one-stage spectrum leasing schemes, reservation-only and on-demand-only, as benchmarks. In the reservation-only scheme as used in [8], the MVNO prescribes a number of SCs at the beginning of a period and has no other chance to purchase spectrum resources throughout the entire period. The reservation-only problem can be formulated similarly to the stochastic programming in (2), except that $\mathbf{E}_{\mathcal{X},c_{\mathrm{s}}}\left[Q\left(\mathcal{X}, c_{\mathrm{s}}, n_{\mathrm{r}}\right)\right]$ is replaced by $\mathbf{E}_{\mathcal{X}}\left[G\left(\mathcal{X}, n_{\mathrm{r}}\right)\right]$, which is irrelevant to the on-demand stage. The optimal reservation, denoted by n_{ro}^*, is then given by

$$n_{\mathrm{ro}}^* = \nabla U^{-1}\left(\frac{c_{\mathrm{r}}}{u_{\mathrm{g}}\mathbf{E}_{\mathcal{X}}\left[\Theta(\mathcal{X})\right]}\right). \tag{41}$$

The average surplus can be computed accordingly. In the on-demand-only scheme as used in [12], the MVNO purchases spectrum resources dynamically according to the on-demand users and their locations. The on-demand-only problem is a special case of the optimization problem (3), where $n_{\mathrm{r}} = 0$. The optimal on-demand request, denoted by n_{so}^*, is given by

$$n_{\mathrm{so}}^* = \nabla U^{-1}\left(\frac{c_{\mathrm{s}}}{u_{\mathrm{g}}\Theta(\mathcal{X})}\right). \tag{42}$$

The surplus averaged over the period can be calculated accordingly.

We compare the surpluses achieved by the proposed two-stage leasing scheme with those by the one-stage leasing schemes under different on-demand price variation ξ_{c_s} and traffic variation ξ_K in Fig. 7a and 7b, respectively. Since the reservation-only scheme is irrelevant to the on-demand price, the corresponding line remains constant for different ξ_{c_s} in Fig. 7a. From both figures, we can see that the two-stage leasing scheme achieves much higher surplus than that of reservation-only scheme for all ξ_{c_s} and ξ_K. The differences becomes more apparent as ξ_{c_s} or ξ_K increases. This is because that the on-demand stage preserves flexibility to acquire additional SCs to deal with emerging traffic. In this way, two-stage leasing scheme effectively reduces overbooking in the advance reservation stage. Moreover, we can see that the two-stage leasing scheme also outperforms on-demand-only scheme for all ξ_{c_s} and ξ_K. The advantage is more distinct as ξ_{c_s} or ξ_K becomes smaller. This is because that the first stage enables the MVNO to reserve SCs at a low cost. With the reserved SCs to serve a baseline amount of traffic, the MVNO can avoid purchasing expensive on-demand SCs in real time. In sum, the two-stage leasing scheme can take advantage of both the low cost of advance reservation and the flexibility of on-demand request.

VII. Conclusions

In this paper, we derived a two-stage leasing scheme, which enables the MVNO to take advantage of both the low cost of advance reservation and the flexibility of on-demand request. To find the optimal leasing decisions in the two stages, we formulated the problem as a tri-level nested optimization problem, and solved the problem efficiently. With our analysis, the MVNO can easily calculate the optimal amount of SCs to lease in each stage. Numerical results showed that the derived two-stage spectrum leasing scheme adapts to different levels of network variations, and achieves more surplus than conventional one-stage leasing schemes.

References

[1] C. Liang and F. R. Yu, "Wireless network virtualization: A survey, some research issues and challenges," *IEEE Commun. Surveys Tuts.*, vol. 17, no. 1, pp. 358–380, 1st Quart., 2015.

[2] F. Granelli *et al.*, "Software defined and virtualized wireless access in future wireless networks: Scenarios and standards," *IEEE Commun. Mag.*, vol. 53, no. 6, pp. 26–34, Jun. 2015.

[3] S. Bi, R. Zhang, Z. Ding, and S. Cui, "Wireless communications in the era of big data," *IEEE Commun. Mag.*, vol. 53, no. 10, pp. 190–199, Oct. 2015.

[4] K. Samdanis, X. C. Perez, and V. Sciancalepore, "From network sharing to multi-tenancy: The 5G network slice broker," *IEEE Commun. Mag.*, vol. 54, no. 7, pp. 32–39, Jul. 2016.

[5] M. Richart, J. Baliosian, J. Serrat, and J.-L. Gorricho, "Resource slicing in virtual wireless networks: A survey," *IEEE Trans. Netw. Service Manage.*, vol. 13, no. 3, pp. 462–476, Sep. 2016.

[6] Y. Zaki, L. Zhao, C. Goerg, and A. Timm-Giel, "LTE wireless virtualization and spectrum management," in *Proc. 3rd Joint IFIP Wireless Mobile Netw. Conf. (WMNC)*, Oct. 2010, pp. 1–6.

[7] R. Kokku, R. Mahindra, H. Zhang, and S. Rangarajan, "NVS: A substrate for virtualizing wireless resources in cellular networks," *IEEE/ACM Trans. Netw.*, vol. 20, no. 5, pp. 1333–1346, Oct. 2012.

[8] T. Guo and R. Arnott, "Active LTE RAN sharing with partial resource reservation," in *Proc. IEEE 78th Veh. Technol. Conf. (VTC Fall)*, Sep. 2013, pp. 1–5.

[9] M. I. Kamel, L. B. Le, and A. Girard, "LTE wireless network virtualization: Dynamic slicing via flexible scheduling," in *Proc. IEEE 80th Veh. Technol. Conf. (VTC Fall)*, Sep. 2014, pp. 1–5.

[10] L. Chen, F. R. Yu, H. Ji, G. Liu, and V. C. M. Leung, "Distributed virtual resource allocation in small-cell networks with full-duplex self-backhauls and virtualization," *IEEE Trans. Veh. Technol.*, vol. 65, no. 7, pp. 5410–5423, Jul. 2016.

[11] L. Duan, J. Huang, and B. Shou, *Cognitive Virtual Network Operator Games* (SpringerBriefs in Computer Science). New York, NY, USA: Springer, 2013.

[12] D. H. N. Nguyen, Y. Zhang, and Z. Han, "A contract-theoretic approach to spectrum resource allocation in wireless virtualization," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Dec. 2016, pp. 1–6.

[13] K. Zhu and E. Hossain, "Virtualization of 5G cellular networks as a hierarchical combinatorial auction," *IEEE Trans. Mobile Comput.*, vol. 15, no. 10, pp. 2640–2654, Oct. 2016.

[14] J. Chase, R. Kaewpuang, W. Yonggang, and D. Niyato, "Joint virtual machine and bandwidth allocation in software defined network (SDN) and cloud computing environments," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Jun. 2014, pp. 2969–2974.

[15] N. C. Luong, P. Wang, D. Niyato, Y. Wen, and Z. Han, "Resource management in cloud networking using economic analysis and pricing models: A survey," *IEEE Commun. Surveys Tuts.*, vol. 19, no. 2, pp. 954–1001, 2nd Quart., 2017.

[16] Y. Zhang, S. Bi, and Y. J. A. Zhang, "A two-stage spectrum leasing optimization framework for virtual mobile network operators," in *Proc. IEEE Int. Conf. Commun. Syst. (ICCS)*, Dec. 2016, pp. 1–6.

[17] *Evolved Universal Terrestrial Radio Access (E-UTRA), Further Advancements for E-UTRA Physical Layer Aspects Version 9.2.0*, 3rd Generation Partnership Project, document TR. 36.814, Mar. 2017.

[18] G. Song and Y. Li, "Utility-based resource allocation and scheduling in OFDM-based wireless broadband networks," *IEEE Commun. Mag.*, vol. 43, no. 12, pp. 127–134, Dec. 2005.

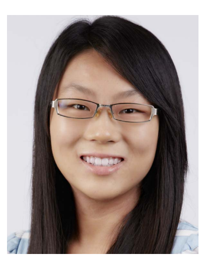
[19] R. B. Kellogg, T. Y. Li, and J. Yorke, "A constructive proof of the Brouwer fixed-point theorem and computational results," *SIAM J. Numer. Anal.*, vol. 13, no. 4, pp. 473–483, 1976.

[20] X. Wang and G. Giannakis, "Resource allocation for wireless multiuser OFDM networks," *IEEE Trans. Inf. Theory*, vol. 57, no. 7, pp. 4359–4372, Jul. 2011.

[21] J. Huang and L. Gao, *Wireless Network Pricing* (Synthesis Lectures on Communication Networks). San Rafael, CA, USA: Morgan Claypool, Jun. 2013.

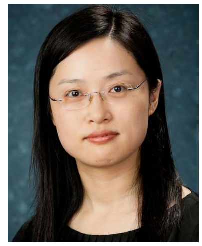
[22] E. Hazan, "Introduction to online convex optimization," *Found. Trends Optim.*, vol. 2, nos. 3–4, pp. 157–325, 2016.

[23] Y. J. Zhang and S. C. Liew, "Proportional fairness in multi-channel multi-rate wireless networks—Part II: The case of time-varying channels with application to OFDM systems," *IEEE Trans. Wireless Commun.*, vol. 7, no. 9, pp. 3457–3467, Sep. 2008.



Yingxiao Zhang received the B.Eng. degree in information engineering from The Chinese University of Hong Kong, Hong Kong, in 2012, where she is currently pursuing the Ph.D. degree. Her research interests are wireless network virtualization, cloud radio access networks, and cooperative communications.

Suzhi Bi (M'13) received the B.Eng. degree in communications engineering from Zhejiang University, Hangzhou, China, in 2009, and the Ph.D. degree in information engineering from The Chinese University of Hong Kong, Hong Kong, in 2013. He was a Research Engineer Intern with the Institute for Infocomm Research, Singapore, and a Visiting Student with the EDGE Laboratory, Princeton University, Princeton, NJ, USA, in 2010 and 2012, respectively. From 2013 to 2015, he was a Research Fellow with the Department of ECE, National University of Singapore, Singapore. He is currently an Associate Professor with the College of Information Engineering, Shenzhen University, Shenzhen, China. His research interests include wireless information and power transfer, wireless resource allocation and medium access control, and smart power grid communications. He was a co-recipient of the IEEE SmartGridComm Best Paper Award in 2013. He is currently an Associate Editor of the IEEE ACCESS.



Ying-Jun Angela Zhang (S'00–M'05–SM'11) received the Ph.D. degree in electrical and electronic engineering from the Hong Kong University of Science and Technology, Hong Kong, in 2004.

Since 2005, she has been with the Department of Information Engineering, The Chinese University of Hong Kong, where she is currently an Associate Professor. She was with the Wireless Communications and Network Science Laboratory, Massachusetts Institute of Technology, in 2007 and 2009, and with the Netlab, California Institute of Technology, in 2014. Her research interests include mainly wireless communications systems and smart power systems, in particular optimization techniques for such systems.

Dr. Zhang was a recipient of the 2014 IEEE ComSoc APB Outstanding Paper Award, the 2013 IEEE SmartgridComm Best Paper Award, the 2011 IEEE Marconi Prize Paper Award on Wireless Communications, the Young Researcher Award from The Chinese University of Hong Kong, in 2011, and the Hong Kong Young Scientist Award 2006, conferred by the Hong Kong Institution of Science. She has served as a Workshop Chair of the IEEE ICCC 2014 and 2013, the TPC Vice Chair of Wireless Networks and Security Track of the IEEE VTC 2014, the TPC Vice-Chair of Wireless Communications Track of the IEEE CCNC 2013, the TPC Co-Chair of Wireless Communications Symposium of the IEEE GLOBECOM 2012, the Publication Chair of the IEEE TTM 2011, the TPC Co-Chair of Communication Theory Symposium of the IEEE ICC 2009, the Track Chair of ICCCN 2007, and the Publicity Chair of the IEEE MASS 2007. She was the Co-Chair of the IEEE ComSoc Multimedia Communications Technical Committee and the IEEE Communication Society GOLD Coordinator. She is an Executive Editor of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. She is also an Associate Editor of the IEEE TRANSACTIONS ON COMMUNICATIONS. She served many years as an Associate Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS and *Security and Communications Networks* (Wiley), and a Guest Editor of a feature topic in the *IEEE Communications Magazine*.